

Gravitational Microlensing in NUT Space

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ABSTRACT

We study the theoretical signature of magnetic masses on the light curve of gravitational microlensing effect in NUT space. The light curves for microlensing events in NUT space are presented and contrasted with those due to lensing produced by normal matter. In the next step, associating magnetic mass to massive astrophysical compact objects (MACHOs), we try to see its effect on the light curves of microlensing candidates observed by the MACHO group. Presence or absence of this feature in the observed microlensing events can shed light on the question of the existence of magnetic masses in the Universe.

Key words: : gravitational lensing – relativity – cosmology: Observations – Cosmology: theory –dark matter.

1 INTRODUCTION

Studying the rotational curves of spiral galaxies gives the most important evidence for the existence of dark matter in the galactic halo (Faber & Gallagher 1979 ; Trimble 1987). Results from 21 cm band observation shows that for thousands of spiral galaxies, rotational curves remains constant beyond their luminous radius (Persic et al. 1996). Comparing luminous matter of universe $\Omega_{lum} = 0.004$ (Fukugita et al. 1995) with the amount of baryonic matter $\Omega_B = 0.02h^{-2}$ (obtained from nucleosynthesis models of universe) confirms that a major part of the halo consists of baryonic dark matter (Copi et al.

1995; Burles & Tyler 1998). One of the possible forms of baryonic dark matter in the halo could be massive astrophysical compact halo objects (MAHCOs), which are obscure owing to their light mass. Black holes and Neutron stars can also be considered in this category. The pioneering idea of using the gravitational microlensing technique for detection of MACHOs was proposed by Paczyński (1986). Since his proposal, gravitational microlensing theory entered into its observational phase with work by several groups. In this paper we study the gravitational microlensing in an exotic space-time, called NUT space (Newman, Unti & Tamburino 1963). The usual gravitational lens effect is based on the bending of light rays passing a point mass M in Schwarzschild space-time. In the paper of Nouri-Zonoz & Lynden-Bell (1997) the gravitational lens effect on light rays passing by a NUT hole has been considered using the fact that all the geodesics in NUT space including the null ones lie on cones. It is shown that compared with the Schwarzschild lens, there is an extra shear due to the gravitomagnetic field that shears the shape of the source. The effect is shown to be small even for big values of the magnetic masses (NUT factor). In this paper we will obtain the gravitational microlensing light curves in NUT space and compare with the observational light curves of a few dozen microlensing candidates. The outline of the paper is as follows. In section 2, we give a brief account on the results of gravitational macrolensing by NUT space and then in the third section we discuss the microlensing on light rays by NUT space and, in particular, we find the magnification of a point-like star. In section 4 we use observational microlensing light curves, taken by MACHO collaboration, toward the Large Magellanic Cloud (LMC) and the Galactic bulge to fit with theoretical light curves in NUT space. In the final section, the fitting results are analyzed.

2 GRAVITATIONAL MACROLENSING IN NUT SPACE

The metric of NUT space is given (in t, r, θ, ϕ coordinates) by the line element

$$ds^2 = f(r) (dt - 2l \cos\theta d\phi)^2 - \frac{1}{f(r)} dr^2 - (r^2 + l^2) (d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where $f(r) = 1 - \frac{2(Mr+l^2)}{r^2+l^2}$ and l is called the magnetic mass or NUT factor and one can think of $Q = 2l$ as the strength of the gravitomagnetic monopole represented by the NUT solution (Lynden-Bell & Nouri-Zonoz 1998). It was shown that all the geodesics of NUT space, including the null ones, lie on a cone for which the semi-angle is given by

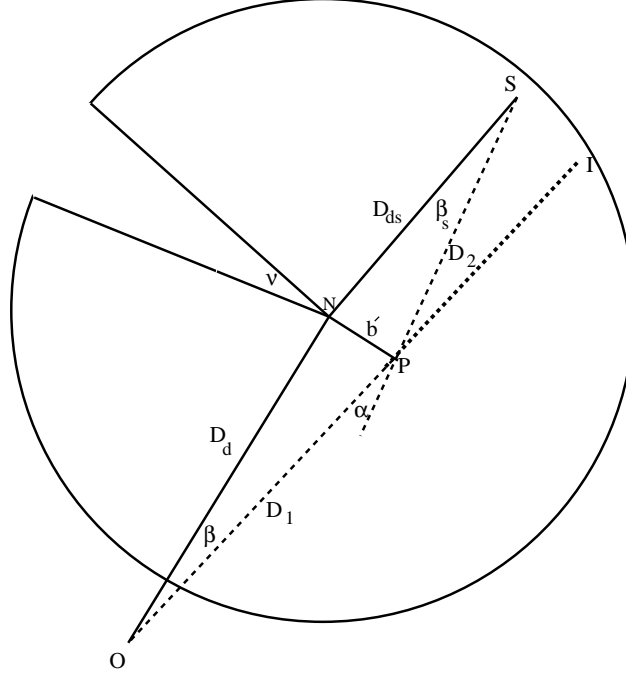


Figure 1. Open, flattened cone and the light ray (dashed line) which is deflected at P passing the NUT lens. $\nu = 2\pi(1 - \frac{L}{(L^2 + \epsilon^2 Q^2)^{1/2}})$ and α are the deficit and bending angles respectively.

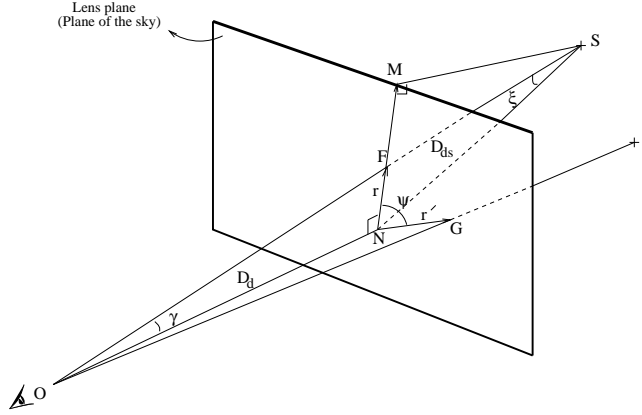


Figure 2. Lens plane and the position of source S , image I and observe O .

$$\sin \chi = \frac{Q}{b[1 + Q^2/b^2]^{1/2}},$$

where b is the impact parameter defined on the cone (Nouri-Zonoz & Lynden-Bell 1997). The geometry of lensing in the case of NUT space could be shown in the following two figures. In Fig.1 the path of a light ray deflected at point P is shown on an open flattened cone and in Fig.2 the positions of the source and image are shown on the lens plane. The relation between the positions of source and image is given by

$$\frac{r}{r'} = \frac{[4\chi^2 + (\alpha - \bar{\beta})^2]^{1/2}}{\bar{\beta}}, \quad (2)$$

where, r and r' denote the positions of source and the image respectively, $\bar{\beta} = \beta(1 + \frac{D_d}{D_{ds}})$ with the parameters β , D_d and D_{ds} as defined in Fig. 1 and $\alpha = 4Gm/bc^2$ is the bending angle defined on the cone. Using the Jacobian of transformation between image and source positions, the magnification of the image is given by the following relation:

$$A = \frac{1}{[(1 - \alpha^2/\bar{\beta}^2)(1 - \alpha^2/\bar{\beta}^2 - 8\chi^2/\bar{\beta}^2)]^{1/2}} \quad (3)$$

It can easily be seen that for $\chi = 0$ one recovers the known result of the Schwarzschild lens. It is shown that for an extended source the orientation of the image is also dependent on the NUT factor through the definition of χ (Lynden-Bell & Nouri-Zonoz 1997; 1998). Using the above results we study the microlensing by NUT space in the next section.

3 GRAVITATIONAL MICROLENSING IN NUT SPACE

In this section we introduce the basics of gravitational microlensing by a Schwarzschild lens and then study the same effect in NUT space.

3.1 Basics of gravitational microlensing

Considerable gravitational lensing occur when the impact parameter of the light rays is small enough. Since in gravitational microlensing the deflection angle is too small, for present telescopes it is impossible to resolve the two images produced and its effect is only on the magnification of background star. This magnification is given by:

$$A(t) = \frac{u(t)^2 + 2}{u(t)\sqrt{u(t)^2 + 4}}, \quad (4)$$

where $u(t) = \sqrt{u_0^2 + (\frac{t-t_0}{t_E})^2}$ is the impact parameter (position of the source in deflector plane normalized by Einstein radius) and in which t_E is the Einstein crossing time (duration of event) defined by $t_E = R_E/v_t$, where v_t is the transverse velocity of deflector with respect to the line of sight. The Einstein radius is given by $R_E^2 = \frac{4GMD}{c^2}$, where M is the mass of the deflector and $D = \frac{D_d D_{ds}}{D_s}$. It is seen that the light curve is symmetrical with respect to time and since gravitational lensing effect is independent of the frequency of light, we would expect the same magnification throughout the spectrum. The probability of observing a microlensing event is very low (e.g. toward the Large and Small Magellanic Clouds is about 10^{-7}) and the rate of microlensing events also depends on the galactic models (Rahvar 2003). Comparing the rate of

observed microlensing with what have been expected from theoretical galactic halo models reveal that only 20 per cent of the halo is made of MACHOs (Lasserre et al. 2000; Alcock et al. 2000).

3.2 Gravitational Microlensing in NUT space

In the Galactic scales the configurations of gravitational lensing have dynamical behavior and this makes gravitational microlensing light curves very sensitive to the parameters of the space-time under consideration. In what follows, we find the magnification function for gravitational microlensing in NUT metric and compare it with Schwarzschild microlensing. Using Eq. (3) for the magnification of a point like source in NUT space, it can be written in the following form:

$$A(u_i) = \left[\left(1 - \frac{1}{u_i^4}\right) \left(1 - \frac{1}{u_i^4} - \frac{8R^4}{u_i^4}\right) \right]^{-\frac{1}{2}} \quad (5)$$

where $R = \frac{R_{NUT}}{R_E} = c\sqrt{\frac{l}{2GM}}$ and u_i indicates the position of the image in the lens plane (normalized to Einstein radius). In which we define the NUT radius to be

$$R_{NUT}^2 = 2lD. \quad (6)$$

Here we are interested in obtaining the magnification of the background star as a function of the position of the source in the lens plane in the absence of the lens. Using definitions of the Einstein and NUT radii and normalizing all the length scales to Einstein radius, Eq.(2) can be written in the following form:

$$u_s^2 = 4\frac{R^4}{u_i^2} + \left(\frac{1}{u_i} - u_i\right)^2, \quad (7)$$

where, u_s and u_i are the positions of the source and the image on the lens plane respectively. Hereafter, we omit the index s of u_s for convenience. In principle, Equation (7) has the following two solutions:

$$\frac{1}{u_i^{\pm 2}(u)} = \frac{1 + u^2/2 \pm \sqrt{(1 + u^2/2)^2 - (4R^4 + 1)}}{4R^4 + 1}, \quad (8)$$

corresponding to the positions of the two images produced by the lens provided $u^2 > 2(\sqrt{1 + 4R^4} - 1)$. Now using Eq.(5) the magnification for each of the images can be written in the following form:

$$A^{\pm} = \frac{\left(1 - \frac{8R^4}{u_i^{\pm 4} - 1}\right)^{-1/2}}{\left|1 - \frac{1}{u_i^{\pm 4}}\right|}, \quad (9)$$

Substituting equation (8) into equation (9), the total magnification is:

$$A(u) = |A^-| + |A^+| \quad (10)$$

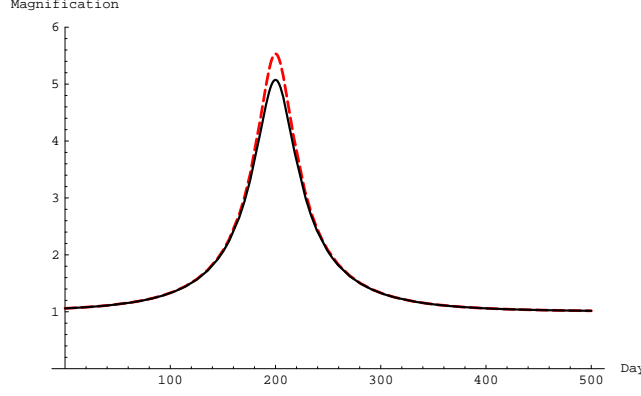


Figure 3. Solid line represents the gravitational microlensing in Schwartzschild metric with the parameters of $u_0 = 0.2$, $t_E = 100$ and $t_0 = 200$ and dashed line shows the light curve in NUT space with $R = 0.2$, $u_0 = 0.2$, $t_E = 100$ and $t_0 = 200$ parameters. The magnification of light in NUT space is more than Schwartzschild space with the same impact parameter.

$$= \frac{1}{\left(1 - \frac{(2+u^2 - \sqrt{-16R^4 + 4u^2 + u^4})^2}{4(1+4R^4)^2}\right) \sqrt{1 - \frac{8R^4}{-1 + \frac{4(1+4R^4)^2}{(2+u^2 - \sqrt{-16R^4 + 4u^2 + u^4})^2}}}} - \frac{1}{\left(1 - \frac{(2+u^2 + \sqrt{-16R^4 + 4u^2 + u^4})^2}{4(1+4R^4)^2}\right) \sqrt{1 - \frac{8R^4}{-1 + \frac{4(1+4R^4)^2}{(2+u^2 + \sqrt{-16R^4 + 4u^2 + u^4})^2}}}},$$

where we use the fact that A^+ is negative for $u^2 > 2(\sqrt{1+4R^4} - 1)$. For $R = 0$ one can recover Paczyński's relation, Eq. 4. Expanding equation (10) in terms of R^4 we obtain the following simple expression for the magnification:

$$A(u) = \frac{2+u^2}{u\sqrt{4+u^2}} + \frac{8R^4(2+u^2)}{u^3(4+u^2)^{3/2}} + \mathcal{O}(R^8) + \dots \quad (11)$$

In Fig. 3 the gravitational microlensing light curves in NUT and Schwartzschild spaces are shown. In the next section we use realistic light curves of microlensing candidates towards the Large Magellanic Cloud and the Galactic bulge to test their compatibility with theoretical light curves in NUT space.

4 COMPATIBILITY OF MICROLENSING IN NUT SPACE WITH OBSERVATIONS

In this section we compare the light curves of microlensing candidates, observed by MACHO collaboration, with those obtained (in the previous section) from our theoretical study of light curves in NUT space. 44 microlensing light curves towards the Galactic bulge (1997a) and LMC (1997b) have been analysed. These data have been obtained from MACHO group's database available on the net[★]. In order to increase the sensitivity of fitting to the light curves, we express the light curves in terms of the magnification rather than

[★] <http://www.macho.mcmaster.ca/>

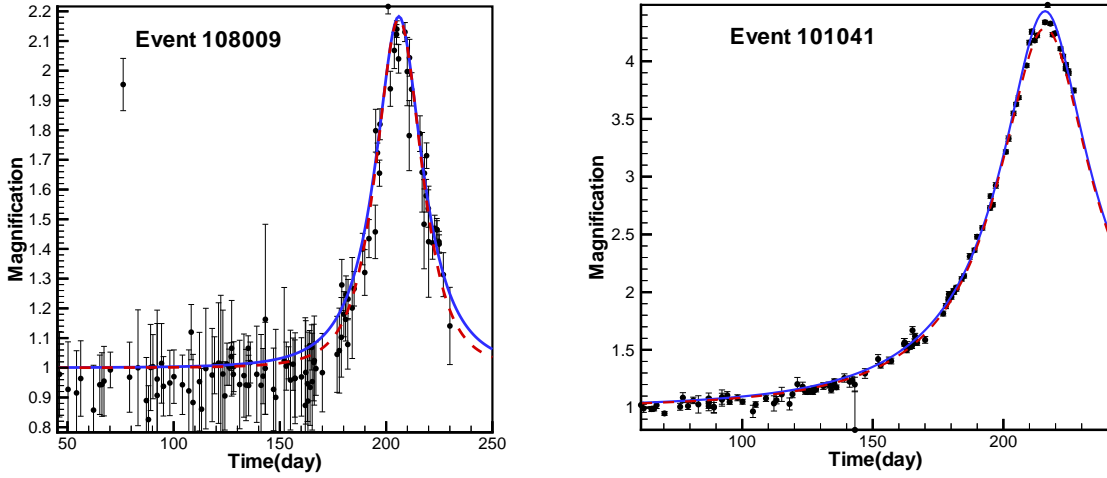


Figure 4. Left and right panels show the observed light curves of events (108009) and (101041) by MACHO collaboration. The best fit of gravitational microlensing light curves are indicated in NUT (dashed line) and Schwarzschild (solid line) spaces.

magnitude of the background star. Tables (1) and (2) show the results of our analysis of fitting data with the light curves in the Schwarzschild and NUT metrics. In the Table (1), it is seen that including the NUT charge do not improve the fitting parameter $\chi^2/N_{d.o.f}$, but in some cases like lmc1b, lmc7 and lmc8 we obtain a non-zero value for the R . This can be interpreted through the degeneracy problem that arises in the fitting. For two event in the Galactic bulge candidates, fitting is improved by the inclusion of the NUT factor. In the event (101041), χ^2 is about 453/105_{d.o.f} from NUT fitting, while in the Schwarzschild space χ^2 is 570/106_{d.o.f}. The reconstructed parameter of magnetic mass from this fit is $R = 0.41$ with an uncertainty of 0.031 from covariance matrix. In the second event (108009) the goodness of fitting to NUT is weaker than the event (101041) and the value of χ^2 is 278/118_{d.o.f} in the NUT space compared to 282/119_{d.o.f} in the Schwarzschild metric. Fig. (4) shows the light curves of the microlensing candidates (101041) and (108009) with the best fitting in NUT and the Schwarzschild spaces. to the LMC and galactic bulge stars respectively.

To test the reliability of NUT fitting to the light curve of event (101041), we also tried to fit this light curve with another well known effect, the so-called non-standard microlensing light curve in the Schwarzschild metric. Since the duration of this event is long enough, the parallax effect should be taken into account. If the variation of the velocity rotating component of the Earth around the sun is not negligible with respect to the projected transverse speed of the deflector, then the apparent trajectory of the deflector with respect to the line of sight is a cycloid instead of a straight line. The resulting ampli-

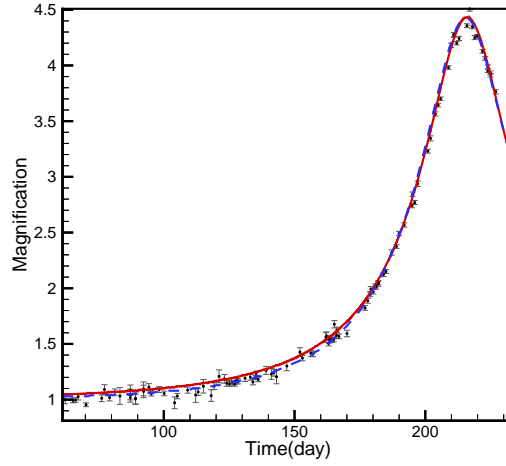


Figure 5. Dashed-line and solid line indicate the best fit to the light curve of event (101041) using the parallax effect and standard microlensing respectively. The reconstructed parameters including the parallax effect are obtained to be: $u_0 = 0.36$, $t_0 = 221\text{day}$, $t_E = 64$, $\tilde{v} = 0.08 A.U./\text{day}$ and $\theta = -0.65\text{Rad}$.

Table 1. light curve of microlensing candidates towards LMC have been fitted with microlensing in Schwarzschild and NUT space. The result of fit is shown with the best χ^2 and reconstructed parameters

<i>Event :</i>	χ^2_{NUT}	$N_{d.o.f}^{NUT}$	u_0^{NUT}	t_0^{NUT}	t_E^{NUT}	R^{NUT}	χ^2_{Sch}	$N_{d.o.f}^{Sch}$	u_0^{Sch}	t_0^{Sch}	t_E^{Sch}
lmc 1a	1841	495	0.13	56	17	0.0044	1845	496	0.13	56	17
lmc 1b	412	389	0.12	57	16	0.3	421	390	0.12	57	17
lmc 4	861	261	0.34	646	19	0.031	864	262	0.34	646	19
lmc 5	209	264	0.025	24	27	0.0004	206	265	0.025	25	27
lmc 6	321	397	0.44	197	50	0.01	322	398	0.44	197	50
lmc 7	832	266	0.67	463	27	0.81	838	267	0.2	463	51
lmc 8	315	261	0.89	389	27	0.68	317	262	0.51	389	34

cation versus time curve is therefore affected by this parallax effect (Grieger *et al* 1986; Gould 1992). Analysing this event, taking into account the parallax effect, shows the goodness of fit $\chi^2 = 356/104_{d.o.f}$ which is compared to the standard microlensing in Fig.(5). Here we obtain the transverse speed of deflector on the ecliptic plane $\tilde{v} = \frac{v}{1-x} = 0.08 \text{ au d}^{-1}$. It is seen that the parallax effect improves the fit more than the inclusion of the magnetic mass (NUT factor).

5 SUMMARY

In this article we have studied the gravitational microlensing in NUT space with the aim of learning more concerning the magnetic masses and their observability regarding MACHOs. In the first step we introduced the ratio

Table 2. light curve of microlensing candidates towards Galactic Bulge have been fitted with microlensing in Schwarzschild and NUT space. The result of fit is shown with the best χ^2 and reconstructed parameters

<i>Event :</i>	χ^2_{NUT}	$N_{d.o.f}^{NUT}$	u_0^{NUT}	t_0^{NUT}	t_E^{NUT}	R^{NUT}	χ^2_{Sch}	$N_{d.o.f}^{Sch}$	u_0^{Sch}	t_0^{Sch}	t_E^{Sch}
101001	1143	103	0.08	160	24	0.00044	339	104	0.12	161	26.7
101041	453	105	0.35	216	64	0.41	570	106	0.23	216	71
101044	554	104	0.18	203	14	0.	547	105	0.18	203	14
101046	112	107	0.017	177	6.22	0.	112	108	0.01	177	6.23
104013	154	110	1.65	161	4.23	0.98	154	111	0.77	161	6.8
104036	58	96	0.31	103	13	0.	58	97	0.31	103	13.5
104037	4510	104	0.18	116	89	0.26	4692	105	0.14	116	92
108009	275	118	1.67	206	10	1.23	282	119	0.5	206	23
108024	134	108	0.35	203	20	0.0003	116	109	0.36	203	20
108054	1621	111	0.05	196	9.7	0.031	1621	112	0.05	196	9.7
110003	91	80	0.44	94	4.89	0.0054	91	81	0.44	94	4.8
110008	50	77	0.59	107	5.6	0.0044	50	78	0.6	107	5.6
110011	74	60	0.34	129	9.7	0.00008	74	61	0.34	129	9.7
110055	44	76	0.49	166	10.8	0.031	44	77	0.49	166	10.8
111029	52	53	0.42	177	5.9	0.07	52	54	0.42	177	6
111039	34	55	0.4	254	58	0.054	34	56	0.40	254	58
113026	91	80	0.44	94	5	0.	91	81	0.44	94	4.8
113014	310	110	0.67	161	7.6	0.	154	111	0.77	161	6.8
113023	139	146	0.32	188	8.8	0	139	147	0.32	188	8.8
114021	111	116	0.83	168	27	0	111	117	0.83	168	27
114042	223	110	0.27	181	8.27	0	223	111	0.27	181	8.27
115026	201	64	0.38	87	16	0.1	201	65	0.38	87	16
118026	102	117	0.38	77	11.8	0.031	102	118	0.28	77	11.8
118038	100	118	0.49	80	24	0.04	100	119	0.49	80	24
119001	double Lens										
119005	90	118	0.46	215	6.8	0.01	90	119	0.46	215	6.8
119053	133	105	3.45	156	4.63	2.21	136	106	0.53	156	17
120007	28	71	2.41	161	2.97	2.12	33	72	0.25	161	14.3
121026	206	71	0.18	94	13	0.028	206	72	0.18	94	13
124002	248	63	0.00004	8.3	40	1.24	226	64	0.02	42	68
124031	72	64	0.02	168	16	0.17	34	65	0.000002	167	15
128027	234	96	0.14	221	8.34	0	234	97	0.14	221	8.34
128055	328	98	0.49	143	15	0.044	328	99	0.49	143	15
159053	362	40	0.17	130	25	0.26	367	41	0.13	130	26
162006	283	46	0.36	145	11	0.044	283	47	0.36	145	11
167004	73	45	0.33	108	18	0	72	64	0.33	108	18

$R = \frac{R_{NUT}}{R_E}$ which in a sense is the ratio of magnetic mass of the lens to its mass. Then we found the magnification for the microlensing in NUT space in terms of this parameter. In the next step comparing the light curves in NUT space with a set of observational light curves from MACHO collaboration. We showed that the inclusion of magnetic masses in the form of the NUT metric as the surrounding space-time of the lens, instead of the usual Schwarzschild lens in most cases will not change the fit. Even in those cases where the fit is changed one finds that the fit would be much better with the inclusion of the parallax effect.

In the above considerations we have assumed that there are magnetic mass constituents of MACHOs and studied their observability through microlensing effect. In a recent paper (Bradley et al 1999) global solutions composed of locally perfect fluids that are matched with the NUT metric and interpreted

as its source are presented. Owing to the fact that mass of MACHOs could range from that of a large planet to a few M_{\odot} , we do not see any argument against the conjecture that the fluid cores discussed in that paper could be probable candidates for MACHOs.

Although the above consideration shows that the effect of NUT factor is almost negligible but one can not rule out the existence of NUT charge on that basis. As a matter of fact with the next generation of microlensing experiments in which both the photometric precision and sampling will be improved, the (non)-existence of NUT charge could be addressed with much more precision. A Monte-Carlo simulation for observability of magnetic masses in the next generation microlensing experiments is in progress.

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